

Constitutive Observability and the Measurement of Prospective Health

State Identification and Prospective Health under Maintenance Constraints

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Paper VI in the series: *Health, Formally Defined*.

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Abstract

A prospective Health measurement seeks to determine whether an organization realizes its specified constitution and possesses adequate capacity for lawful continuation. We examine this problem in a completely specified two-site exchange model. Exact present flux and constitutive cost leave a two-state observation fiber whose members have different future viability under common forcing. An informative maintained intervention can alter the prospective property being measured. We construct a heterogeneous-response probe that identifies the complete fiber through post-probe flux while preserving each prospective classification, and prove local authorization under bounded baseline, intervention, readout and future-law uncertainty. We also establish a fixed-drive alternative that identifies the same states through repeated cost observation, preserves their classifications, and admits a separate nonzero local robustness region. The comparison distinguishes sensing channels, experimental assumptions and certified tolerances without assigning physical superiority to either procedure. Continuous-time analysis establishes a work-budget obstruction for the corresponding exact-flow specification and exhibits an independent intermediate survival failure between qualified observations. These results provide a model-relative account of informative, constitutively maintained and prospectively preserving measurement. They clarify the role of a measured organizational state in subsequent Health analysis while leaving physical state imaging, calibration and prospective prediction subject to independent empirical validation.

1 Introduction

A measurement of prospective Health concerns both an organization’s present condition and the continuations available to it under a specified set of requirements. The *Health, Formally Defined* series develops a definition in which constitutional realization and sufficient lawful continuation capacity jointly determine the property of interest [1, 2, 3, 4, 5]. To evaluate that property from measurement, the available observations must resolve distinctions that affect the specified Health question.

The preceding papers establish a connected sequence of scientific obligations. Paper I formulates the constitutive continuation-capacity definition; Paper II specifies contextual licensing and lawful transport; Paper III characterizes query-sufficient representations and the information needed for their transport; Paper IV examines information provenance and conservative refinement in a finite organizational model; and Paper V studies empirical identification, uncertainty, and predictive-law calibration in an auditable forest example. The present paper studies an experimentally different question: which interventions can resolve an initially hidden state distinction while maintaining the organization and preserving the prospective answer to be measured? These contributions jointly identify the mathematical, informational and empirical responsibilities of prospective Health measurement.

The defining proposition is

$$H_{S,q}(z; d, t) \iff R_S(z) \wedge Adeq_{S,q}(Cap_S(z; d, t)). \quad (1)$$

The specification, context, horizon and adequacy query jointly fix the intended scientific property.

We study a system in which its present flux and cost can be measured accurately while a consequential part of its state remains unresolved. The states compatible with those readings satisfy the same present constitution but respond differently to a common future condition. An additional experiment can resolve the ambiguity, provided its effects on the system are included in the analysis. We therefore examine identification, constitutive maintenance during measurement, and preservation of the subsequent prospective Health classification as distinct properties of an intervention.

The two-site construction permits an exhaustive description of every state compatible with the initial measurements. Its specified dynamics give a unique future endpoint, allowing viability to be tested against the full set of constitutive conditions. We compare two maintained identifying interventions that use different post-intervention observation channels, and establish decision guarantees under errors in the baseline observations, uncertain intervention parameters, and locally varying future forcing. A continuous-time counterpart identifies how a different dynamical and maintenance specification can change the answer. A separate trajectory exhibits a survival failure between qualified observations.

The work draws on observability, functional observability, the comparison of experiments, and partial identification [16, 17, 18, 21, 19, 20, 22, 12, 11]. Its contribution is the joint treatment of a specified prospective property, the full information fiber of a measurement, the effect of an identifying intervention on that property, and the conditions required for a reliable decision. The construction is deliberately finite and analytically transparent. Exact rational identities prove the representative cases. The uncertainty bounds follow from explicit rational enclosures, with the governing interval rules and decision margins given in the appendices.

We first specify the constitutive and prospective questions, then determine which state distinctions

are visible in the present measurements. The analysis proceeds from a measurement-induced change in prospective Health to two preserving identification procedures with distinct sensing requirements and bounded-error guarantees. We subsequently examine response-capacity transport, continuous-time behavior, and the interpretation of an organization’s state as the measurand of a future state-imaging system. The appendices collect exact arithmetic, interval bounds, and concise formal-verification provenance.

2 Prospective Health and the measurement problem

2.1 State, continuation, and the property of interest

Let X denote the space of possible states under the scientific model. The specification $\mathcal{S}$ identifies the organization and the conditions defining its realization, the dynamics governing possible histories, and the responses through which continuation capacity is represented. A requirement q determines whether the resulting capacity is adequate. The state, future context, observation horizon, and requirement together determine the prospective property in Eq. (1). A measurement provides information about that state through an observation map or an experimentally justified statistical model.

For an application to a material system, the state description must also have an identified physical target, observational support, units where appropriate, and reference conditions. Those choices establish the meaning of the property being investigated. Evidence from the measurement determines which states remain compatible with the observations. Whether this information is sufficient for a given prospective question can be tested directly on the resulting compatible-state sets.

Definition 2.1 (Fixed-query sufficiency). For a target query $H : X \rightarrow \text{Prop}$ and observation $M : X \rightarrow Y$, write $\text{Sufficient}(M, H)$ if

$$M(x) = M(x') \implies [H(x) \leftrightarrow H(x')] \quad \text{for all } x, x' \in X. \quad (2)$$

1

Different scientific questions require different levels of state information. Measurements may determine a numerical capacity while leaving another constituent of the state unresolved. They may also determine an adequacy decision while remaining insufficient to reconstruct the complete capacity response. Each claim of observational sufficiency therefore specifies the property to be inferred. The example below makes the difference between present-state discrimination and prospective discrimination explicit.

2.2 Existence, reliability, and robustness of continuation

Continuation capacity admits several scientifically distinct evaluations. Existence asks whether at least one lawful history preserves the constitution. Reliability assigns a measure to viable histories under a specified distribution of future conditions. Robustness asks whether the required

¹Formal counterpart:

`Tier33.RelationalHealth.Observation.observationHealthSufficient_iff_kernel_refines.`

continuation remains available as disturbances or controls vary. The distinction between a control selected after observing a disturbance, $\forall d \exists u$, and a control selected in advance, $\exists u \forall d$, becomes essential whenever the order of action and uncertainty matters.

For a finite family of possible histories $\mathcal{H}_x$ and a normalized distribution p_x over them, the viable mass is

$$m_V(x; p_x) = \sum_{\omega \in \mathcal{H}_x} p_x(\omega) \mathbf{1}_{\text{Viable}(x, \omega)}. \quad (3)$$

Its reserve-qualified analogue uses the intersection of viability and reserve satisfaction. The complete probability-specialized query is

$$H_{p, \theta}(x) = R_S(x) \wedge [m_V(x; p) \geq \theta]. \quad (4)$$

The future distribution and adequacy threshold are part of the scientific question. Different distributions can assign different degrees of reliability to the same viable-history set. Existence of a viable history and sufficient probability of viable continuation consequently support different prospective conclusions.

A complete probabilistic interpretation distinguishes variation in future histories, uncertainty about the present state, and variation between repeated measurements. For example, a state may have viable-continuation mass $3/4$ under a specified future distribution while current evidence assigns probability $1/2$ to that state. These numbers describe different uncertainties. All probabilities used in the present construction belong to mathematical model distributions; empirical distributions would require separate estimation and calibration.

2.3 From observation to a prospective decision

The Health proposition yields a truth-valued answer for a specified state and requirement. Its constituent capacities may carry numerical values, such as viable-history mass, that can be estimated from observations under a suitable physical model. A measurement procedure first produces an indication and a set of compatible states; a prospective decision follows when the required answer is common to all those states. Physical measurement additionally requires a validated relationship among the measurand, reference, indication, and uncertainty [6, 7, 9].

3 A finite exchange model with constitutive maintenance

3.1 Exchange, cost, and qualification

Consider a network of finitely many sites V , each carrying a rational-valued profile $\phi : V \rightarrow \mathbb{Q}$. Symmetric exchange weights $w_{ij} = w_{ji} \geq 0$ describe coupling between sites, while positive coefficients a and b set exchange and local relaxation. We use rational coefficients, drives, durations, and thresholds throughout the sampled-time analysis so that every stated numerical comparison is exact. The exchange flux and constitutive cost are

$$J_{ij}(\phi) = aw_{ij}(\phi_i - \phi_j), \quad (5)$$

$$C(\phi) = \frac{a}{2} \sum_{i, j \in V} w_{ij}(\phi_i - \phi_j)^2 + b \sum_{i \in V} \phi_i^2. \quad (6)$$

The functional C combines gradient and absolute-profile terms and is a declared mathematical cost. Its coefficients have no established physical energy or entropy interpretation in this study.

Fix a nonempty required edge family E_{req} , positive operating and survival floors o, s with $s \leq o$, maintenance budget $B \geq 0$, maximum recovery steps N , and recovery time $T > 0$. A state qualifies when

$$\text{Qualified}(\phi) \iff (\forall (i, j) \in E_{\text{req}} : |J_{ij}(\phi)| \geq o) \wedge [C(\phi) \leq B]. \quad (7)$$

The survival predicate applies the lower exchange floor to all required edges while retaining the same cost budget:

$$\boxed{\text{Survives}(\phi) \iff (\forall (i, j) \in E_{\text{req}} : |J_{ij}(\phi)| \geq s) \wedge [C(\phi) \leq B].} \quad (8)$$

Put $(L_w \phi)_i = \sum_j w_{ij}(\phi_i - \phi_j)$ and $E = aL_w + bI$. A sampled evolution has states $\phi^0, \dots, \phi^n$, prescribed drives $u^0, \dots, u^{n-1}$, and a positive rational step $h \in \mathbb{Q}_{>0}$ satisfying

$$\phi^{k+1} = \phi^k + h(u^k - E\phi^k), \quad h \left(b + 2a \sum_j w_{ij} \right) \leq 1 \quad (i \in V). \quad (9)$$

The target shares the source network and parameters. Each drive is subject to the source-work budget

$$|\langle u^k, \phi^k \rangle| \leq B \quad (0 \leq k < n). \quad (10)$$

For a sampled history $\gamma = (\phi^0, \dots, \phi^n; u^0, \dots, u^{n-1}; h)$, the complete maintenance requirement is

$$\boxed{\begin{aligned} \text{Maintains}(\gamma) \iff & [1 \leq n \leq N] \wedge [h > 0] \wedge [nh \leq T] \\ & \wedge \text{Qualified}(\phi^0) \wedge \text{Qualified}(\phi^n) \\ & \wedge [\forall k \in \{0, \dots, n\} : \text{Survives}(\phi^k)] \\ & \wedge [\forall k < n : \phi^{k+1} = \phi^k + h(u^k - E\phi^k)] \\ & \wedge [\forall k < n : |\langle u^k, \phi^k \rangle| \leq B] \\ & \wedge [\forall i \in V : h \left(b + 2a \sum_j w_{ij} \right) \leq 1]. \end{aligned}} \quad (11)$$

For a specified one-step drive and duration, $\text{Maintains}(z, z')$ abbreviates this predicate for its corresponding history. Survival concerns recorded states; continuous-time behavior between observations is examined separately in Section 11.

3.2 Unforced contraction and finite survival failure

Write $\lambda = 1 - bh$. Under the stability inequality in Eq. (9), $0 \leq \lambda < 1$.

Theorem 3.1 (Recorded unforced contraction). *For a wholly unforced admissible sampled evolution with $\|\phi^0\|_\infty \leq M$ and $M \geq 0$, every recorded $k \leq n$ satisfies*

$$\|\phi^k\|_\infty \leq \lambda^k M, \quad |J_{ij}(\phi^k)| \leq 2aw_{ij}\lambda^k M. \quad (12)$$

If a required edge has positive survival floor s and the history reaches an index k for which $2aw_{ij}\lambda^k M < s$, the recorded state at k fails survival. Such an index exists for any arbitrarily extendable unforced stable history.²

Proof. The unforced Euler update at site i is

$$\phi_i^{k+1} = (1 - h(b + a \sum_j w_{ij}))\phi_i^k + ha \sum_j w_{ij}\phi_j^k.$$

All coefficients are nonnegative under the stated stability bound, and their sum equals λ . Thus $\|\phi^{k+1}\|_\infty \leq \lambda \|\phi^k\|_\infty$. Induction proves the first inequality. The triangle inequality in J_{ij} proves the second. Once the upper bound is strictly below s , the survival requirement fails. Geometric decay gives a finite crossing index; for a particular finite history that index must be included among its recorded samples. $\square$

3.3 Maintained and nonmaintained two-site histories

Take $V = \{0, 1\}$, $w_{01} = w_{10} = 1$, $a = b = 1$, $o = 2$, $s = 1$, $B = 9$, $N = 2$, $T = 1$, and $h = 1/6$. Then

$$J_{01}(x, y) = x - y, \quad C(x, y) = (x - y)^2 + x^2 + y^2. \quad (13)$$

For this two-site specialization the variable state is the profile $z = (z_0, z_1) \in X_0 := \mathbb{Q}^2$; network, parameters, required edge and operating thresholds remain fixed. Any additional fields in the formal maintained-gradient representation are determined by these fixed data and the profile. The realization predicate and existential adequacy specialize explicitly to

$$R_S(z) \iff \text{Qualified}(z), \quad \text{Adeq}_\exists(K) \iff K \neq \emptyset. \quad (14)$$

The existential capacity collects endpoints of lawful maintained histories from z under the allowed control class. Thus the subsequent Health equivalence follows from the displayed predicates and the stationary-history construction, with no further realization assumption.

The profile $\phi = (1, -1)$ qualifies with flux magnitude 2 and cost 6. Its Euler operator is $E\phi = (3, -3)$.

One stationary continuation uses $n = 1$, constant profiles $(1, -1)$, drive $u = (3, -3)$, and $h = 1/6$. Balance is exact; source and target qualify; the stable coefficient is $h(b + 2aw_{01}) = 1/2$; elapsed time is $1/6$; flux and cost remain 2 and 6; and the drive pairing has absolute value $6 \leq 9$.

A two-step recovery has $n = 2$, profiles

$$(1, -1) \longrightarrow (1/2, -1/2) \longrightarrow (1, -1) \quad (15)$$

with drives $(0, 0)$ and $(9/2, -9/2)$. The flux magnitudes are $(2, 1, 2)$, costs $(6, 3/2, 6)$, and absolute drive pairings $(0, 9/2)$. Both endpoint qualifications hold, each sample survives, and the recovery has duration $1/3$ and two steps, within the declared limits. The corresponding exact inequalities are collected with the mathematical verification materials in Appendix B.

For the requirement that at least one maintained continuation be available, the stationary history establishes positive prospective Health for $(1, -1)$. The unforced trajectory provides a contrasting example: an evolution can satisfy its dynamical equation while crossing the survival threshold. Together these histories show why lawful dynamics and maintenance must be evaluated separately.

²Formal counterpart: `Tier33.RelationalHealth.Measurement.Exchange.unforced_edge_bound`.

Remark 3.2 (Existential continuation under stationary control). For the two-site model, every presently qualified state admits a stationary maintained continuation under unrestricted control. Present qualification therefore determines the existential Health query (Theorem 5.1). The same construction applies on a finite symmetric exchange network whenever the stationary drive $u = E\phi$ is permitted: its work satisfies $\langle E\phi, \phi \rangle = C(\phi)$, and an admissible positive duration completes the maintained history. Restrictions on future forcing, available control, or reliability lead to more discriminating prospective questions. Such restrictions become central to the analysis below.

The unforced two-step continuation visits $(1, -1), (1/2, -1/2), (1/4, -1/4)$ and reaches flux $1/2 < s$ at its final recorded sample. It is a lawful sampled trajectory and fails the maintained-continuation requirement.

4 Observational ambiguity under exchange symmetry

4.1 A hidden common-mode degree of freedom

For $c \in \mathbb{Q}$, define the common-mode translation $T_c\phi = \phi + c\mathbf{1}$, where $\mathbf{1}_i = 1$. Directly from Eq. (5),

$$J_{ij}(T_c\phi) = J_{ij}(\phi), \quad \sum_j J_{ij}(T_c\phi) = \sum_j J_{ij}(\phi). \quad (16)$$

Full edge flux and net divergence therefore have an observational symmetry. The absolute-profile term in C need not share it.

Proposition 4.1 (Exact constitutive and complete-Health collision). *Under the fixed two-site constitution of Section 3, let $\phi = (1, -1)$ and $\psi = (3, 1) = T_2\phi$. Their complete directed edge fluxes and full net divergences agree, while*

$$C(\phi) = 6, \quad C(\psi) = 14. \quad (17)$$

*At budget $B = 9$, ϕ has the complete Health property and ψ fails it. Thus the complete flux observation is insufficient for that fixed Health query.*³

Proof. Both profiles have coordinate difference 2, so their directed fluxes are $(J_{01}, J_{10}) = (2, -2)$; net divergence is identical. Equation (13) yields $C(\phi) = 4 + 1 + 1 = 6$ and $C(\psi) = 4 + 9 + 1 = 14$. The first state qualifies and has the stationary maintained continuation shown in Section 3; hence it satisfies the nonempty-capacity Health query. The second exceeds the fixed maintenance budget and fails the realization conjunct of Eq. (1). Equal flux observations consequently belong to distinct Health-equivalence classes. $\square$

³Formal counterpart:

`Tier33.RelationalHealth.Measurement.Exchange.flux_observation_full_health_insufficient.`

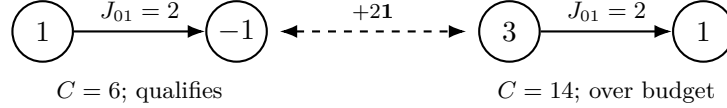


Figure 1: Equal exchange flux can arise from distinct absolute profiles. Their differing constitutive costs lead to different qualification and prospective Health answers under the specified budget.

5 Information sufficient for a specified Health question

5.1 The existential Health question

A measurement can determine a specific prospective property even when it leaves the underlying state partially unresolved. In the two-site model, stationary control makes a maintained continuation available to every qualified state, so the existential Health decision depends on present qualification. A measurement retaining exchange and cost can therefore answer that question without reconstructing each coordinate of the state. The more discriminating future-forcing question is considered in Section 6.

Theorem 5.1 (Fixed-constitution qualification and complete Health). *For every maintained-gradient state g with the original fixed two-site constitution and existential stationary-maintenance observer,*

$$H(g) \iff \text{Qualified}(g).$$

4

Proof. The forward implication follows from the explicit realization predicate in Eq. (14). For the converse, qualification supplies the operating fluxes and cost budget, and the stationary Euler drive provides an admissible positive step with unchanged profile. Both endpoints qualify, every sample survives, and work equals the nonnegative coercive profile cost, bounded by the budget. One recovery step and its stable duration satisfy the original limits. This yields a lawful maintained history and a nonempty collected continuation response, which satisfies the declared existential adequacy requirement. $\square$

This equivalence is specific to the admitted stationary pump, the cost bound on its work, and the one-step recovery permissions. The positive and negative observation fibers below are distinguished by present qualification. Under a restricted or shared available-control family, a thresholded future-history law, or a stronger disturbance query, present qualification need not establish prospective adequacy. Section 6 establishes such a separation under a distinct, shared constrained-future law, leaving the original existential stationary-maintenance query and its sufficiency theorem intact.

5.2 A measurement that identifies Health but leaves states unresolved

Recall $X_0 = \mathbb{Q}^2$, with network, parameters and observation structure fixed as above. Define

$$M_Q(g) = ((J_{ij}(g))_{i,j \in V}, C(g)). \quad (18)$$

⁴Formal counterpart: `Tier33.RelationalHealth.Measurement.HMS03.complete_health_iff_qualified`.

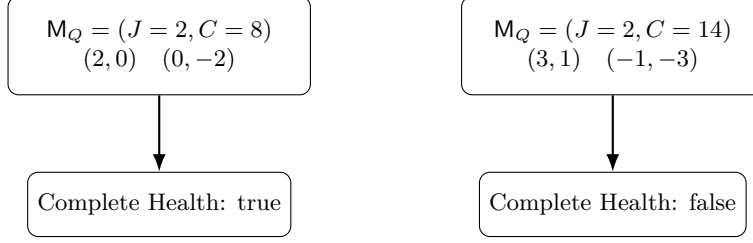


Figure 2: Query-relative observation fibers can contain distinct complete states while remaining homogeneous for the requested Health predicate. The positive and negative fibers are obtained from actual fixed-model profiles and constitutive costs.

This observation is mathematical: it assumes access to complete flux and constitutive cost as declared outputs. No physical calibration of the cost channel is asserted.

Theorem 5.2 (Complete-Health sufficiency with noninjective observation). *On the full two-site state space X_0 , M_Q is sufficient for the complete-Health query, while M_Q is not injective on X_0 :*

$$\text{Sufficient}(M_Q, H) \quad \wedge \quad \neg \text{Injective}(M_Q). \quad (19)$$

5

Proof. Equal M_Q outputs give equality of every required edge-flux magnitude and equality of constitutive cost. Thus the qualified predicates agree. Theorem 5.1 converts equality of qualification answers into equality of existential Health answers. To show noninjectivity, take the distinct profiles

$$\phi = (2, 0), \quad \psi = (0, -2).$$

Both have directed flux $(J_{01}, J_{10}) = (2, -2)$, diagonal flux zero, and cost $C = 8$. Their fixed-model maintained-gradient states are distinct because their profiles differ. They share M_Q and both have complete Health by stationary maintenance, proving Eq. (19). $\square$

The complete-Health observation is sufficient because the qualification clauses and admissible stationary continuation determine that query. This example specializes familiar functional observability; the distinct future-law capacity question is developed in Section 6.

The observation retains meaningful negative fibers as well. Profiles $(3, 1)$ and $(-1, -3)$ have the same directed flux and cost $14 > 9$, so both fail complete Health. Cost alone is not sufficient: the zero profile has cost zero and fails the positive required operating flux. Baseline flux alone is not sufficient by Proposition 4.1.

5.3 Measurement design for a specified target

The criterion for sufficiency is that indistinguishable measurements yield the same answer to the scientific question:

$$M(x) = M(y) \implies (H(x) \iff H(y)). \quad (20)$$

⁵Formal counterpart: `Tier33.RelationalHealth.Measurement.HMS03.query_sufficient_not_injective`.

Recovering the entire state would ensure that any state-dependent query can be evaluated. For a particular question, it is enough that all states sharing the same observation also share the same answer. This is the familiar principle of functional or target observability [21, 22], applied here to a prospective property defined through qualified states and maintained continuations.

6 Prospective capacity under a common future law

The existential question considered above allows any suitable stationary drive. We now examine what happens when the future forcing is specified in advance and applied equally to every candidate state. This question probes the ability to preserve the same constitution under a common future condition.

6.1 Deterministic future evolution and viable capacity

For two-site states $z \in X_0$, prescribe a single future step with duration and forcing

$$h_f = \frac{1}{4}, \quad u_f = \left(\frac{29}{4}, \frac{5}{4}\right), \quad F_f(z) = z + h_f(u_f - Ez). \quad (21)$$

Every state evolves through the same one-step balance equation, giving the endpoint $F_f(z)$. A viable history must also meet the previously defined requirements for constitutive maintenance. The capacity representation consists of endpoints reached by histories satisfying both the dynamics and maintenance conditions:

$$\begin{aligned} \text{Cap}_{\mathcal{L}_f}(z) &= \{\text{endpoint}(\gamma) : \gamma \text{ lawful and maintained under } \mathcal{L}_f\}, \\ \text{H}_{\mathcal{L}_f}(z) &\iff \text{Qualified}(z) \wedge [\text{Cap}_{\mathcal{L}_f}(z) \neq \emptyset]. \end{aligned} \quad (22)$$

The prospective Health answer asks whether the source qualifies and whether any maintained history reaches an endpoint under this forcing. Its value is therefore determined by the states and histories of the model.

More generally, for a single specified constant drive $u \in \mathbb{Q}^2$ and duration $0 < h \leq 1/3$, with one step and elapsed time within the declared recovery limits, set $F_{h,u}(z) = z + h(u - Ez)$. The constitutive realization, unique endpoint, and nonempty-endpoint adequacy reduce the model-relative Health question to

$$\boxed{\text{H}_{\mathcal{L}_{h,u}}(z) \iff \text{Qualified}(z) \wedge \text{Qualified}(F_{h,u}(z)) \wedge |\langle u, z \rangle| \leq 9.} \quad (23)$$

The stability factor is $3h \leq 1$, the intermediate recorded states consist only of these qualified endpoints, and their survival conditions follow because the survival floor is one. Equation (24) is the specialization of Eq. (23) to h_f, u_f .

Lemma 6.1 (Uniqueness and scalar viability criterion). *Every lawful future history from a fixed profile z has endpoint $F_f(z)$. Its prospective Health query is characterized by*

$$\begin{aligned} \text{H}_{\mathcal{L}_f}(z) &\iff \left[|z_0 - z_1| \geq 2 \wedge C(z) \leq 9 \right] \\ &\quad \wedge \left[|F_f(z)_0 - F_f(z)_1| \geq 2 \wedge C(F_f(z)) \leq 9 \right] \wedge [|\langle u_f, z \rangle| \leq 9]. \end{aligned} \quad (24)$$

Proof. The law fixes source, network, parameters, one positive step, duration and drive. The Euler balance therefore forces the endpoint to be $F_f(z)$. Its stability factor is $3h_f = 3/4 \leq 1$; its single step and elapsed time satisfy the recovery limits. Qualification at both recorded samples implies the lower survival floor and cost budget. The remaining maintenance requirements are precisely the three conjunctions displayed: source qualification, endpoint qualification and the prescribed initial-state drive-work bound. A maintained history is a viable continuation, and the explicit one-step construction supplies the converse. All law-admitted histories share the endpoint, so an over-budget endpoint cannot be rescued by selecting another history. $\square$

6.2 Identical present readings and different prospective states

Choose

$$x = \left(\frac{11}{8}, -\frac{7}{8}\right), \quad y = \left(\frac{7}{8}, -\frac{11}{8}\right). \quad (25)$$

Both states have the same directed fluxes $J_{01} = 9/4$, $J_{10} = -9/4$, the same exchange gradients and boundary support, and the same cost $247/32$. They qualify at present and satisfy $H_\exists$.

Theorem 6.2 (Prospective separation within a qualified observation fiber). *For the shared specified law and constitution,*

$$M_Q(x) = M_Q(y), \quad \text{Cap}_{\mathcal{L}_f}(x) = \emptyset, \quad \text{Cap}_{\mathcal{L}_f}(y) = \{F_f(y)\}, \quad \neg H_{\mathcal{L}_f}(x) \wedge H_{\mathcal{L}_f}(y). \quad (26)$$

Consequently, present flux and cost do not determine Health under the specified future forcing, even though they determine the existential stationary-control Health question. ⁷

Proof. The actual future endpoints are

$$F_f(x) = \left(\frac{73}{32}, \frac{7}{32}\right), \quad F_f(y) = \left(\frac{61}{32}, -\frac{5}{32}\right).$$

Both have forward flux $33/16 > 2$. Their respective future drive-work magnitudes are $71/8$ and $37/8$, both at most nine. The endpoints differ in cost:

$$C(F_f(x)) = \frac{4867}{512} > 9, \quad C(F_f(y)) = \frac{4051}{512} < 9.$$

Lemma 6.1 excludes every maintained future history from x and constructs a maintained history from y . Endpoint uniqueness gives an empty response capacity for x and the asserted singleton for y . Equality of present observations with opposite Health values proves insufficiency by the fixed-query kernel criterion. $\square$

⁶Formal counterpart: `Tier33.RelationalHealth.Measurement.HMS05.fixed_future_scalar_criterion`.

⁷Formal counterpart: `Tier33.RelationalHealth.Measurement.HMS04.prospective_separation`.

6.3 Prospective separation through common-mode forcing

Write $\phi_{\pm} = (A \pm m, -A \pm m)$ and prescribe a common drive $(v + d, v - d)$ with step h . The present flux and cost satisfy

$$J_{01}(\phi_{\pm}) = 2A, \quad C(\phi_{\pm}) = 6A^2 + 2m^2. \quad (27)$$

The future contrast and common modes are

$$A' = (1 - 3h)A + hd, \quad m'_{\pm} = (1 - h)(\pm m) + hv.$$

Substitution gives the exact identity

$$\boxed{C(F\phi_+) - C(F\phi_-) = 8h(1 - h)mv.} \quad (28)$$

Proposition 6.3 (Parameter-class prospective separation). *Suppose both $\phi_{\pm}$ qualify, the shared one-step evolution is admissible, the negative-common-mode branch meets its endpoint exchange and cost requirements and drive-work bound, while the positive-common-mode branch has endpoint cost strictly above 9. Then the positive branch has empty registered viable capacity and the negative branch has nonempty viable capacity. This conclusion applies to every rational parameter tuple satisfying those hypotheses.*⁸

Proof. The general one-step maintenance criterion reduces the positive capacity to successful endpoint qualification and work. The over-budget endpoint makes that capacity empty by lawful endpoint uniqueness. The opposite branch's hypotheses give a maintained witness. Equation (28) identifies the mechanism responsible for separating the two endpoint costs; $0 < h < 1$, $m > 0$ and $v > 0$ yield strict ordering. Exchange, work and time conditions remain necessary independently of that identity. $\square$

For the fixed baseline parameters $A = 9/8$, $m = 1/4$, $h = 1/4$, and $d = 3$, the entire rational forcing interval $4 \leq v \leq 9/2$ satisfies these hypotheses. Indeed,

$$\begin{aligned} A' &= \frac{33}{32}, \quad m'_{\pm} = \frac{4v \pm 3}{16}, \quad C(F\phi_{\pm}) = \frac{3267 + 4(4v \pm 3)^2}{512}, \\ C(F\phi_+) &\geq \frac{4711}{512} > 9, \quad C(F\phi_-) \leq \frac{4167}{512} < 9, \\ |J_{01}(F\phi_{\pm})| &= \frac{33}{16} > 2, \quad |\langle (v + 3, v - 3), \phi_{\pm} \rangle| = \left| \frac{27}{4} \pm \frac{v}{2} \right| \leq 9. \end{aligned} \quad (29)$$

The first cost is increasing and is bounded below at $v = 4$; the second is increasing and bounded above at $v = 9/2$. Both sources already qualify, and $3h = 3/4 \leq 1$ with one step inside the recovery limits. This proves the whole sufficient interval; global optimization of the control set remains open. The present observation is invariant under $m \mapsto -m$. Shared future forcing can distinguish the two prospective Health answers.

⁸Formal counterpart: `Tier33.RelationalHealth.Measurement.HMS05.general_prospective_separation`.

7 Complete observation fibers and measurement disturbance

7.1 Exhaustive reconstruction from flux and cost

The ambiguity can be characterized for every state in the two-site model. Introduce the contrast and common-mode coordinates

$$A(z) = \frac{z_0 - z_1}{2}, \quad m(z) = \frac{z_0 + z_1}{2}, \quad R(z_0, z_1) = (-z_1, -z_0).$$

Then $z = (A + m, -A + m)$ and $Rz = (A - m, -A - m)$. Equation (27) shows that two profiles have equal complete flux-and-cost observations exactly when their A coordinates agree and their squared common modes agree.

Theorem 7.1 (Complete fixed-model observation fibers). *For every state of the specified two-site system,*

$$\boxed{M_Q^{-1}(M_Q(z)) = \{z, Rz\}.} \quad (30)$$

*If $m(z) = 0$, this is a singleton. For the observation of Eq. (25), the fiber is exactly $\{x, y\}$.*⁹

Proof. Equality of full directed edge flux determines $A' = A$ by $J_{01} = 2A$. Equal cost then yields $6A^2 + 2m'^2 = 6A^2 + 2m^2$, whence $(m' - m)(m' + m) = 0$ over $\mathbb{Q}$. Thus $m' = m$ or $m' = -m$, giving exactly z or Rz . Both states reproduce the observation. For $J_{01} = 9/4$ and $C = 247/32$, the identities give $A = 9/8$ and $m^2 = 1/16$, hence $m = \pm 1/4$. The stationary inverse reconstructs the fixed-model state from its profile, so the equality holds for the state space, rather than only a hand-picked two-state subtype. $\square$

7.2 An informative intervention that changes prospective Health

An intervention supplies additional information through the system's response. Because the intervention also changes the state, its effect on subsequent viability must be evaluated. Consider the common probe

$$P_\delta(z) = z + \frac{1}{6}(q_\delta - Dz - Ez), \quad D = \text{diag}(1, 0), \quad q_\delta = (4 + \delta, -4). \quad (31)$$

The first studied probe has $\delta = 0$. Its responses for the two initial candidates have distinct edge fluxes and both probe histories satisfy the original sampled **Maintains** predicate. However, the subsequent prospective Health evaluation differs from the initial verdict.

Proposition 7.2 (Actual probe-induced prospective reversal). *For the original probe P_0 ,*

$$\neg H_{\mathcal{L}_f}(x), \quad H_{\mathcal{L}_f}(P_0x).$$

The future reached from the probed negative candidate is

$$F_f(P_0x) = \left(\frac{415}{192}, \frac{5}{48} \right), \quad J_{01} = \frac{395}{192} > 2, \quad C = \frac{54775}{6144} < 9, \quad |\langle u_f, P_0x \rangle| = \frac{479}{64} < 9. \quad (32)$$

¹⁰

⁹Formal counterpart: `Tier33.RelationalHealth.Measurement.HMS05.exact_fiber_set`.

¹⁰Formal counterpart: `Tier33.RelationalHealth.Measurement.HMS05.original_x_verdict_reversal`.

Proof. Substituting $P_0x = (29/24, -49/48)$ into the specified future law yields Eq. (32). For completeness, the two probe histories have the directly calculated quantities

	P_0x	P_0y
$ J_{01} $	107/48	37/16
C	2869/384	1047/128
$ \langle q_0 - Dz, z \rangle $	455/64	527/64

All endpoint fluxes exceed 2, and all endpoint costs and probe source-work magnitudes are at most 9. The initial states qualify; $h = 1/6$ gives $3h = 1/2 \leq 1$, one recovery step and duration $1/6 \leq 1$, and qualified endpoints also survive. Thus both probe histories maintain the original sampled constitution. For the x branch, the probed state is qualified, the specified future endpoint is qualified, and all remaining positive-step, stability, recovery and work clauses hold. Lemma 6.1 proves the positive post-probe Health verdict. The initial negative verdict is Theorem 6.2. The initial-state identification theorem therefore concerns an actual counterfactual state property whose subsequent verdict this probe changes. $\square$

This example demonstrates that a maintained and informative intervention can alter the prospective property under investigation. A useful identifying experiment must therefore be assessed both for the information gained and for the properties of the resulting state.

8 Identification with preservation of prospective Health

8.1 An identifying intervention that maintains the constitution

Keep the same form of intervention and duration, with $\delta = 1/2$ and $q_* = (9/2, -4)$. The rule is common to the two candidate states and specifies a heterogeneous local response, $u(z) = q_* - Dz$. Its numerical action depends on the state actually occupied by the system. A physical implementation would therefore require a calibrated local response to an absolute state reference; the initial flux-and-cost readings alone cannot furnish that reference to a digital controller.

Theorem 8.1 (Full-fiber verdict-preserving experiment). *For $P_* = P_{1/2}$ the probe histories from both x and y satisfy the original Maintains predicate. Their edge-flux responses are distinct by $1/12$, and*

$$\boxed{H_{\mathcal{L}_f}(P_*z) \iff H_{\mathcal{L}_f}(z) \text{ for every } z \in M_Q^{-1}(M_Q(x)).} \quad (33)$$

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Proof. The post-intervention states and their defining quantities are

	x	y
P_*z	$(31/24, -49/48)$	$(23/24, -23/16)$
$J_{01}(P_*z)$	37/16	115/48
$C(P_*z)$	9283/1152	10051/1152
$ \langle q_* - Dz, z \rangle $	499/64	555/64

¹¹Formal counterpart: `Tier33.RelationalHealth.Measurement.HMS05.fiber_verdict_preservation`.

Both initial and challenged states are qualified, and both end costs and drive-work magnitudes lie below 9. The original stability condition is $3h \leq 1$; the stronger heterogeneous-response stability check is

$$h(3 + \max_i \eta_i) \leq 1, \quad \frac{1}{6}(3 + 1) = \frac{2}{3} < 1$$

for $\eta = (1, 0)$. One positive step and duration $1/6$ meet the recovery limits, while qualified endpoints meet the recorded survival floor. Thus both histories **Maintains**. Their flux difference equals $115/48 - 37/16 = 1/12$. Applying the unchanged future law to each challenged state gives

$$F_f(P_*x) = \left(\frac{141}{64}, \frac{1}{8} \right), \quad F_f(P_*y) = \left(\frac{371}{192}, -\frac{1}{6} \right).$$

The costs are $18817/2048 > 9$ and $50179/6144 < 9$, respectively; the corresponding future drive-work magnitudes are $1553/192$ and $989/192$. Both future edge fluxes satisfy the operating floor. Lemma 6.1 proves that the post-probe prospective verdicts are respectively negative and positive. Their original verdicts agree with those signs by Theorem 6.2. The exhaustive fiber characterization of Theorem 7.1 extends the conclusion to every state in the exact observation fiber. $\square$

8.2 Interpretation of the measurement and subsequent future

Preservation compares the same prospective question at two different starting points: the original state z and the state P_*z reached after measurement. The hypothetical future from the original state and the actual future following the intervention have the same specified forcing. The separate trajectories are

$$\begin{aligned} \gamma_p^z : z &\xrightarrow{h_p=1/6} P_*z, \\ \gamma_f^z : z &\xrightarrow{h_f=1/4} F_f(z), \\ \gamma_f^{P_*z} : P_*z &\xrightarrow{h_f=1/4} F_f(P_*z). \end{aligned} \tag{34}$$

The sequence from z through P_*z and then to $F_f(P_*z)$ joins two actual histories at the measured intermediate state. Their durations differ, so each phase is evaluated under its own one-step evolution law. The future from z provides the counterfactual comparison. The post-measurement future is maintained for the positive branch; its negative counterpart exceeds the cost budget. The theorem concerns preservation of the prospective classification under a fixed future query. It allows the detailed endpoint capacity to change, and it places no requirement that the full two-phase trajectory itself remain constituted for either branch.

8.3 A nontrivial interval of preserving interventions

The result extends throughout an interval of common compensation values.

Theorem 8.2 (Compensation interval). *For every rational δ with $1/2 \leq \delta \leq 2/3$, the common rule P_δ maintains both states in the complete fiber of $M_Q(x)$, produces a response-flux separation of $1/12$, and preserves their opposite prospective Health verdicts.* ¹²

¹²Formal counterpart: `Tier33.RelationalHealth.Measurement.HMS05.compensation_interval`.

Proof. The probe endpoint costs and work magnitudes reduce to the exact polynomials

$$\begin{aligned} C(P_\delta x) &= \frac{64\delta^2 + 1320\delta + 8607}{1152}, & C(P_\delta y) &= \frac{64\delta^2 + 1224\delta + 9423}{1152}, \\ W_x(\delta) &= \frac{88\delta + 455}{64}, & W_y(\delta) &= \frac{56\delta + 527}{64}. \end{aligned}$$

All four polynomials are increasing on $[1/2, 2/3]$. At the upper endpoint,

$$\begin{aligned} C(P_{2/3}x) &= \frac{256/9 + 880 + 8607}{1152} < 9, & C(P_{2/3}y) &= \frac{256/9 + 816 + 9423}{1152} < 9, \\ W_x(2/3) &= \frac{(176/3) + 455}{64} < 9, & W_y(2/3) &= \frac{(112/3) + 527}{64} < 9. \end{aligned}$$

The probe fluxes are $107/48 + \delta/6$ and $37/16 + \delta/6$, respectively, and therefore exceed 2. Both initial states qualify. The subsequent future costs are

$$C(F_f(P_\delta x)) = \frac{64\delta^2 + 3320\delta + 54775}{6144} > 9, \quad (35)$$

$$C(F_f(P_\delta y)) = \frac{64\delta^2 + 2904\delta + 48711}{6144} \leq 9. \quad (36)$$

The first future-cost polynomial is increasing and exceeds 9 already at $\delta = 1/2$, where its value is $56451/6144$. The second is increasing and remains below 9 at $\delta = 2/3$, where its value is $(256/9 + 1936 + 48711)/6144$. The future fluxes are $395/192 + \delta/24$ and $133/64 + \delta/24$, both above 2. Their source-work magnitudes are $479/64 + 29\delta/24$ and $291/64 + 29\delta/24$, both below 9 even at $\delta = 2/3$. The original one-step stability, survival and recovery requirements also hold throughout the interval. Endpoint uniqueness makes the negative capacity empty and the positive capacity nonempty. The flux responses equal $107/48 + \delta/6$ and $37/16 + \delta/6$, whose difference is $1/12$. Theorem 7.1 makes the two-state conclusion a uniform statement over the complete exact observation fiber. $\square$

The interval establishes a continuous range of feasible rational compensation settings. The nominal compensation used for the jointly bounded experiment of Section 9 is

$$\delta_c = \frac{55}{12} - 4 = \frac{7}{12} \in \left[\frac{1}{2}, \frac{2}{3} \right], \quad q_{\delta_c} = \left(\frac{55}{12}, -4 \right). \quad (37)$$

Its center therefore belongs to the preserving family of Theorem 8.2. Identifying the full admissible range would require a separate optimization analysis.

8.4 Fixed-drive identification by repeated cost observation

The preceding construction separates the initial two-state fiber through the edge-flux response of a heterogeneous probe. A second experiment is available when the constitutive cost can be measured again after an intervention. It uses the state-independent drive $q_{\text{fix}} = (4, -2)$ and the same positive step duration:

$$\tilde{P}(z) = z + \frac{1}{6}(q_{\text{fix}} - Ez), \quad q_{\text{fix}} = (4, -2). \quad (38)$$

Here the applied drive is identical for every candidate, and the additional observation is $C(\tilde{P}z)$. The comparison therefore distinguishes two choices of post-probe sensor information within the same mathematical constitution.

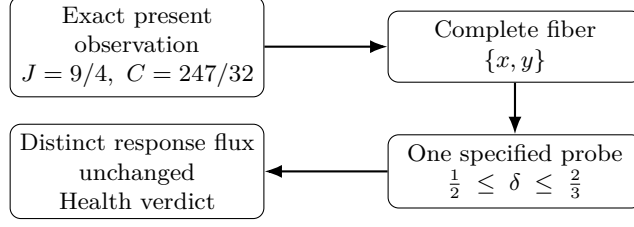


Figure 3: State identification under two observations. Present flux and cost determine the full two-state fiber; the common intervention resolves the remaining prospective distinction while maintaining each state and preserving its Health classification under the specified future law.

Theorem 8.3 (Fixed-drive, cost-readout preserving identification). *The intervention in Eq. (38) maintains the original sampled constitution from both states in Eq. (25), preserves their respective prospective Health verdicts under the common future law, and identifies every state in their exact initial observation fiber through post-probe cost. Specifically,*

$$\tilde{P}x = \left(\frac{23}{16}, -\frac{11}{16}\right), \quad \tilde{P}y = \left(\frac{49}{48}, -\frac{53}{48}\right), \quad (39)$$

$$C(\tilde{P}x) = \frac{903}{128}, \quad C(\tilde{P}y) = \frac{7807}{1152}, \quad (40)$$

$$C(F_f(\tilde{P}x)) = \frac{20071}{2048} > 9, \quad C(F_f(\tilde{P}y)) = \frac{17031}{2048} < 9. \quad (41)$$

Both post-probe fluxes equal $17/8$, whereas their costs differ by $5/18$. Consequently, $C(\tilde{P}z) > 69/10$ identifies x and $C(\tilde{P}z) < 69/10$ identifies y on the exact initial fiber, with opposite preserved prospective Health answers.

Proof. The operator values are $Ex = (29/8, -25/8)$ and $Ey = (25/8, -29/8)$. Substitution into Eq. (38) gives the endpoints in Eq. (39). At both endpoints the flux exceeds 2 and the constitutive cost is below 9; initial states are qualified, and the probe source-work magnitudes are $29/4$ and $25/4$. The step obeys $3h = 1/2 \leq 1$ and satisfies the stipulated duration, survival and recovery requirements. Thus both histories maintain the constitution. The cost difference is

$$\frac{903}{128} - \frac{7807}{1152} = \frac{5}{18}.$$

The future endpoints are $F_f(\tilde{P}x) = (151/64, 21/64)$ and $F_f(\tilde{P}y) = (131/64, 1/64)$. Both have future flux $65/32 > 2$. For the positive y branch, future source work is $289/48 < 9$, and its future endpoint cost is below 9, so its prospective continuation is viable. The x branch exceeds the future cost budget and also violates the future-work budget, since $|\langle u_f, \tilde{P}x \rangle| = 153/16 > 9$; either failure suffices for its negative verdict. These agree with the initial prospective answers by Theorem 6.2. The exact initial fiber is $\{x, y\}$ by Theorem 7.1, completing the identification claim. $\square$

The cost difference is also an instance of the general common-mode separation identity in Eq. (28). For this fixed drive, $h = 1/6$, $m = 1/4$, and $v = (4 - 2)/2 = 1$, hence

$$8h(1 - h)mv = 8 \left(\frac{1}{6}\right) \left(\frac{5}{6}\right) \left(\frac{1}{4}\right) (1) = \frac{5}{18}.$$

The comparator therefore realizes the same common-mode symmetry-breaking mechanism with a cost observation after the intervention.

8.5 Local uncertainty and observation-channel comparison

A direct comparison should account for information acquired after intervention, the bounds on physical control, and the reliability of the requested Health judgment. The heterogeneous probe distinguishes the two states through post-probe flux and already has a quantified joint state/intervention/readout certificate. The fixed-drive experiment separates them through the post-probe constitutive cost, which must be measured with its own uncertainty and reference support. In both experiments, baseline cost remains an assumed available channel.

For a local numerical comparison, let $\rho = 1/4096$ bound independent coordinatewise deviations of the source state from either x or y , fixed-drive duration from $1/6$, both drive coordinates from $(4, -2)$, and the duration and drive coordinates of the subsequent future law from $(1/4, 29/4, 5/4)$. Let the post-probe cost indication have absolute error at most $1/1024$. The following outer enclosures hold uniformly across these rational boxes, with cost measured in the model's own units:

Quantity	Initial branch near x	Initial branch near y
Post-probe flux	[2.123, 2.127]	[2.123, 2.127]
Post-probe cost	[7.047, 7.063]	[6.767, 6.787]
Probe source work	[7.247, 7.253]	[6.247, 6.253]
Subsequent future cost	[9.773, 9.828]	[8.287, 8.345]
Subsequent future flux	[2.026, 2.037]	[2.026, 2.037]

The bounds follow from four-corner rational interval arithmetic applied to the Euler, cost, and source-work expressions. Appendix E states the interval rules used for both experiments. Throughout these boxes the probe is maintained. The future branch near x remains above the cost budget, and the branch near y retains endpoint qualification and future-work admissibility. Allowing the stated cost-readout error, the threshold $69/10$ continues to separate the two branches. This is an independent local mathematical robustness analysis of the fixed-drive alternative. Appendix E records the corresponding universal interval bounds.

The complete noisy-baseline and fixed-drive conclusions can be combined without replacing the original inverse-compatible state family by its nominal centers.

Corollary 8.4 (Complete bounded-error fixed-drive decision). *Let $\rho = 1/4096$, and let Θ_{fixed} consist of all rational probe durations $|h_p - 1/6| \leq \rho$ and drives $\|q_p - (4, -2)\|_\infty \leq \rho$. Let $\mathcal{L}_\rho$ consist of all constant-drive future laws with $|h_f - 1/4| \leq \rho$ and $\|u_f - (29/4, 5/4)\|_\infty \leq \rho$. Suppose that a baseline observation meets the bounds of Theorem 9.1 and its complete compatible family $\mathcal{F}(b, c; e_B, e_C)$ is nonempty. For every*

$$z \in \mathcal{F}(b, c; e_B, e_C), \quad \theta \in \Theta_{\text{fixed}}, \quad \ell \in \mathcal{L}_\rho, \quad |r - C(\tilde{P}_\theta z)| \leq \frac{1}{1024},$$

where $\tilde{P}_\theta z = z + h_p(q_p - Ez)$, the following conclusions hold:

$$\begin{aligned} &\text{Maintains}(z, \tilde{P}_\theta z), \\ &H_\ell(\tilde{P}_\theta z) \iff H_\ell(z), \\ &r < \frac{69}{10} \iff H_\ell(z). \end{aligned}$$

(42)

No binary decision is assigned to an empty compatible family or an indication inconsistent with the admitted error model.

Proof. Theorem 9.1 places every compatible z in one of the two closed coordinate boxes of radius ρ around x and y . The rational enclosure calculation in Appendix D applies to every state in both boxes, every θ and ℓ stated above, and independently bounded cost error. In the x box the probe cost is in $[7.047, 7.063]$ and the post-probe future cost in $[9.773, 9.828]$; in the y box the corresponding costs are in $[6.767, 6.787]$ and $[8.287, 8.345]$. In both boxes source and endpoint qualify and the probe work, duration, stability and recorded survival conditions hold. Before and after probing, the future cost of the x box is above 9, while the y box retains qualified future endpoints and acceptable work. Hence the Health verdict is unchanged in each case. Finally,

$$\frac{6787}{1000} + \frac{1}{1024} < \frac{69}{10} < \frac{7047}{1000} - \frac{1}{1024}$$

separates the entire admitted set of indications. The nonemptiness premise ensures that the rule is applied to an actual compatible candidate, rather than to an empty intersection of measurements. $\square$

The two experiments therefore establish distinct sensing possibilities. The heterogeneous construction provides a proved flux-readout identification theorem with its own jointly quantified uncertainty family. The fixed-drive construction provides a cost-readout identification theorem and a separate bounded-error neighborhood. Neither the recorded error bounds nor the number of sensing channels establishes a physical advantage without calibrated channel uncertainties, actuation constraints, and an experimental comparison of measurement burden. This model establishes the availability of both approaches within their respective declared observation protocols.

8.6 Physical access to the relevant state response

The heterogeneous rule $q - Dz$ is a mathematical state-response law. An instrument implementing it through independently realized local physical relaxation would require its own absolute reference and calibrated response coefficient. A digital controller restricted to the baseline observation can select only $u = G(M_Q(z))$; it therefore selects the same drive for x and y . For the contrast $A(z) = (z_0 - z_1)/2$, homogeneous Euler dynamics give the exact invariance equation

$$A(F_{h,u}z) = (1 - 3h)A(z) + \frac{h}{2}(u_0 - u_1). \quad (43)$$

Because $A(x) = A(y)$ and baseline-only feedback applies the same drive to the two states, $J_{01}(F_{h,u}x) = J_{01}(F_{h,u}y)$. Thus such feedback cannot reproduce the specific heterogeneous flux separation. The obstruction concerns the stipulated output and one-step controller class, and does not exclude alternative sensors or later adaptive experiments. The present constructive result is conditional on an independently realized and calibrated heterogeneous state-response mechanism.

9 Measurement uncertainty and future-law variation

9.1 Recovering the complete compatible-state family

For an indicated baseline flux b and cost c with declared absolute errors e_B, e_C , define the complete compatible family over the two-site state space:

$$\mathcal{F}(b, c; e_B, e_C) = \{z : |J_{01}(z) - b| \leq e_B, |C(z) - c| \leq e_C\}. \quad (44)$$

From $J_{01} = 2A$ one obtains $A \in [(b - e_B)/2, (b + e_B)/2]$. The identity $m^2 = (C - 6A^2)/2$ then produces an outward-safe enclosure of the squared common mode. A positive lower bound retains both sign branches; a negative upper bound rejects the entire family; an interval containing zero must retain the zero-common-mode possibility.

Theorem 9.1 (Noisy baseline certifies the full candidate family). *Suppose the indicated baseline centers lie within $1/262144$ of the nominal readings $9/4$ and $247/32$, and the nonnegative declared error bounds are each at most $1/262144$. Every profile compatible with those measurements lies within coordinatewise radius $\rho = 1/4096$ of x or y . This implication holds on the entire fixed-model state carrier.* ¹³

Proof. The true flux and cost deviate from the nominal values by at most $1/131072$. The resulting rational interval estimates include

$$A \in \left[\frac{294911}{262144}, \frac{294913}{262144} \right], \quad m^2 \in \left[\frac{4292935677}{68719476736}, \frac{4296998909}{68719476736} \right].$$

Here the indicated centers and their allowed observation errors each contribute at most $1/262144$, so with

$$A_{\pm} = \frac{9}{8} \pm \frac{1}{262144}, \quad C_{\pm} = \frac{247}{32} \pm \frac{1}{131072}$$

we have $A_- \leq A \leq A_+$ and, because $A_- > 0$,

$$\boxed{\frac{C_- - 6A_+^2}{2} \leq m^2 \leq \frac{C_+ - 6A_-^2}{2}}. \quad (45)$$

Evaluation yields precisely the two fractions displayed above. The strict squared comparisons

$$\left(\frac{2047}{8192} \right)^2 \leq \frac{4292935677}{68719476736} < \frac{4296998909}{68719476736} \leq \left(\frac{2049}{8192} \right)^2$$

place m in one of $\pm[2047/8192, 2049/8192]$. On either sign branch,

$$|A - 9/8| + ||m| - 1/4| \leq \frac{1}{262144} + \frac{1}{8192} = \frac{33}{262144} < \frac{1}{4096}.$$

As $z = (A + m, -A + m)$, both coordinates lie within $1/4096$ of x or y . These are complete inverse-compatible families, with no omitted negative-sign branch. $\square$

The error limit in Theorem 9.1 is expressed in model units. Physical use requires independent calibration of the attainable error range and of the reference supporting the cost indication. Bounds on timing, heterogeneous coupling, and commanded compensation are separate instrument premises.

9.2 Uniform performance under measurement uncertainty

Let $\rho = 1/4096$ bound independent variations in the two initial-state coordinates, probe duration around $1/6$, the first relaxation coefficient around 1, and the two compensation coordinates around $(55/12, -4)$. The second relaxation coefficient lies in $[0, \rho]$, and the challenged flux has readout error at most $1/1024$. These bounds define a nonempty family of possible states and interventions. The resulting guarantee is uniform over all rational values in that family. Its small numerical tolerances describe a mathematical operating region whose physical feasibility requires separate calibration.

¹³Formal counterpart: `Tier33.RelationalHealth.Measurement.HMS06.baseline_certifies_full_family`.

Theorem 9.2 (Local jointly bounded prospective authorization). *Across every admissible rational state and probe configuration in those boxes, the initial profile qualifies, the actual sampled probe Maintains, and the prospective verdict is unchanged by the probe under the nominal future law. Bounded-error response intervals of the two branches are disjoint. Every compatible indication is assigned its actual prospective Health verdict by the threshold*

$$\boxed{\text{reported flux} > \frac{341}{144} \iff \text{prospectively positive.}} \quad (46)$$

Unsupported indications authorize no binary decision. ¹⁴

Proof. For every allowed input, the exact interval recursion in Eq. (63) gives the full certificate in Table 1. In both branches, the initial flux is at least $2249/1000 > 2$ and the initial cost is at most $7723/1000 < 9$. The probe endpoint fluxes exceed 2, costs are at most $8827/1000 < 9$, and work magnitudes at most $8748/1000 < 9$. With $\rho = 1/4096$, the permitted step interval obeys

$$0 < \frac{1}{6} - \rho \leq h_p \leq \frac{1}{6} + \rho, \quad (\frac{1}{6} + \rho)(4 + \rho) < 1, \quad h_p < \frac{1}{3}, \quad (47)$$

which verifies the strengthened heterogeneous stability test as well as the one-step recovery deadline. Qualified endpoints supply recorded survival. The negative x -branch future costs exceed 9 before and after probing, while in the positive y -branch both future costs are below 9 and the future fluxes exceed 2; its source-work bounds are at most $4628/1000 < 9$ before and $5258/1000 < 9$ after probing. For every accepted flux indication r , the final inequalities

$$r_x \leq \frac{291}{125} + \frac{1}{1024} < \frac{341}{144} < \frac{301}{125} - \frac{1}{1024} \leq r_y$$

complete the two-branch decision. Appendix E provides an intermediate-coordinate ledger and a worked interval propagation through both experiment phases; the enclosing fractions are valid over the full Cartesian input boxes. $\square$

9.3 Robustness to local changes in future conditions

The distinction between the two states is stable under a local variation of the future conditions. Describe a future law by its step duration and two forcing coordinates, and let $\mathcal{L}_\rho$ contain all rational parameter choices within $1/4096$ of the nominal values in Eq. (21). Both candidate branches are evaluated under the same selected law.

Theorem 9.3 (Uniform separation over a local specified future-law family). *For all jointly compatible initial profiles, probe parameters and laws in $\mathcal{L}_\rho$, the branch near x has negative prospective Health and the branch near y has positive prospective Health, both before and after the specified probe. The rule in Eq. (46) identifies the same-law prospective answer uniformly over this family.* ¹⁵

Proof. The future parameter intervals in Eq. (62) vary independently, with the same chosen law used for both branches of any comparison. Table 1 bounds the initial future costs

¹⁴Formal counterpart:

`Tier33.RelationalHealth.Measurement.HMS06.integrated_mathematical_measurement_authority.`

¹⁵Formal counterpart: `Tier33.RelationalHealth.Measurement.HMS06.certified_baseline_law_authority.`

by $[9486/1000, 9525/1000]$ and $[7892/1000, 7932/1000]$, and the post-probe future costs by $[9206/1000, 9262/1000]$ and $[8180/1000, 8235/1000]$, respectively. Thus the x cost exceeds 9 in both epochs. For y , the initial future flux is at least $2059/1000 > 2$, the post-probe future flux at least $2098/1000 > 2$, both endpoint costs are below 9, and the corresponding source-work magnitudes are bounded above by $4628/1000$ and $5258/1000$. These inequalities, together with one-step admissibility and qualified initial and probed sources established in Eq. (47), prove negative and positive viable-capacity outcomes in the respective boxes. The same independently bounded readout inequalities in Theorem 9.2 identify the positive branch for every law in $\mathcal{L}_\rho$. $\square$

9.4 When the evidence supports a decision

A baseline observation may admit states with different prospective outcomes even when the proposed intervention is maintained for every compatible state. Before intervention, the compatible state and law families must be nonempty, and maintenance and classification preservation must hold throughout them. After acquiring the challenged response, a Health conclusion is justified when every remaining compatible state gives the same answer. An empty compatible family indicates inconsistency in the observation model or error bounds; a family with differing Health answers calls for further information. These cases allow experimental permission and diagnostic interpretation to be assessed separately.

The existential query $H_\exists$, the specified-future query $H_{\mathcal{L}}$, and probability-weighted capacity questions evaluate distinct features of continuation. In particular, $\neg(\forall \mathcal{L} H_{\mathcal{L}}(z))$ does not imply $\forall \mathcal{L} \neg H_{\mathcal{L}}(z)$. A policy whose control is chosen after disturbance need not agree with a policy required to be fixed beforehand. These distinctions are part of the formal query, rather than adjustable conventions of a classifier.

10 Preservation of the decision and change in capacity

A stable prospective classification can accompany a change in the future responses available to the organization. In the positive branch, viable capacity consists of one endpoint before measurement and a different endpoint afterward. Preservation of the adequacy answer therefore raises a separate question about how complete response capacities relate across the intervention.

Proposition 10.1 (Response transport and viability requirements). *For the original fixed future map F_f , which is invertible on the two-site state space, any specified probe map P induces a response-coordinate map*

$$T_P = F_f \circ P \circ F_f^{-1}, \quad T_P(F_f z) = F_f(Pz). \quad (48)$$

If viability of the corresponding before/after futures is equivalent, their response-valued capacities satisfy the exact set identity

$$\boxed{\text{Cap}_{\mathcal{L}_f}(Pz) = T_P[\text{Cap}_{\mathcal{L}_f}(z)]}. \quad (49)$$

The commutation identity alone does not imply that capacity correspondence. ¹⁶

¹⁶Formal counterpart: `Tier33.RelationalHealth.Measurement.HMS06.transported_capacity`.

Proof. The affine F_f has nonzero contrast and common-mode multipliers $1 - 3h_f = 1/4$ and $1 - h_f = 3/4$, hence an inverse. Equation (48) follows by cancellation. A nonempty viable future has the unique endpoint $F_f z$, and the corresponding post-probe future has endpoint $F_f(Pz)$. Under equivalence of viability, image of the singleton is the post-probe singleton; when both are nonviable the sets are empty. If viability changes, one capacity can be empty and the other nonempty, so no conjugacy identity on endpoints repairs the missing continuation. Exact correspondence of response capacities consequently requires an explicit relationship between viable histories in addition to the mapping of their endpoints. $\square$

A response transport map must preserve the quantities and adequacy predicates relevant to the scientific query before it licenses a broader capacity or Health equivalence. In the chosen sup metric the nominal future map is $3/4$ -Lipschitz, giving an endpoint-perturbation bound when both capacities are nonempty singletons. Empty/nonempty comparisons and probability-valued policy capacities require separately declared metrics or numerical margins. Neither a response bijection nor the shared Health verdict establishes physical noninvasiveness.

11 Continuous-time interpretation and temporal resolution

The preceding results concern states recorded at discrete times under a rational Euler evolution and a specified source-work rule. A continuous-time formulation also evaluates the intervening trajectory, where constitutive survival and work may vary between observations. We therefore consider an exact real-valued flow for the same exchange operator, with maintenance required throughout the observation interval.

11.1 Exact trajectories and all-time maintenance

For real two-site profiles, define

$$\dot{z}(t) = u(t) - Ez(t), \quad E = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}, \quad C(z) = (z_0 - z_1)^2 + z_0^2 + z_1^2. \quad (50)$$

The nominal heterogeneous compensated probe is

$$\dot{z} = q - (E + D)z, \quad E + D = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}, \quad q = \left(\frac{55}{12}, -4 \right).$$

Its eigenvalue rates are $(5 \pm \sqrt{5})/2$, allowing explicit real-analytic solutions. A continuous history requires an actual differentiable curve satisfying the differential equation, with the prescribed source and terminal state. Continuous maintenance requires source and endpoint qualification, the survival floor and cost bound *at every time*, the recovery deadline and the chosen instantaneous work condition

$$| \langle q - Dz(t), z(t) \rangle | \leq 9 \quad \text{for all } t \in [0, T]. \quad (51)$$

This instantaneous convention differs from the sampled initial-state work pairing in Eq. (10). A cumulative-work convention would define another separately specified continuation requirement.

Continuous prospective law. The continuous-future experiment uses the same constant forcing as the nominal sampled future, now interpreted as the driving term of an exact differential equation. For either original state $p \in \{x, y\}$, write $z_f(t; p)$ for the unique future trajectory defined by

$$\dot{z}_f(t; p) = u_f - Ez_f(t; p), \quad z_f(0; p) = p, \quad u_f = \left(\frac{29}{4}, \frac{5}{4}\right), \quad 0 \leq t \leq T_f = \frac{1}{4}. \quad (52)$$

The constant drive u_f is the actual input throughout this future history; the heterogeneous feedback term Dz belongs only to the preceding probe experiment. The continuous-future constitution requires the source and terminal profiles to satisfy $|z_0 - z_1| \geq 2$ and $C(z) \leq 9$, and every intermediate profile to satisfy $|z_0 - z_1| \geq 1$ and $C(z) \leq 9$. It also requires the recovery deadline $T_f \leq 1$ and the instantaneous work condition

$$|\langle u_f, z_f(t; p) \rangle| \leq 9 \quad \text{for every } t \in [0, T_f].$$

Its response capacity comprises the endpoints of all lawful trajectories satisfying these conditions, and adequacy means that this capacity is nonempty. These conditions define the continuous-future Health question independently of the sampled future and of the continuous heterogeneous probe.

11.2 A work-budget obstruction under exact dynamics

Theorem 11.1 (Nontransfer to the specified exact-flow, all-time-work model). *The nominal y probe with $T = 1/6$, $D = \text{diag}(1, 0)$ and $q = (55/12, -4)$ satisfies the sampled maintenance predicate. Its unique exact continuous-time solution violates the specified instantaneous-work budget at the endpoint:*

$$|\langle q - Dz(T), z(T) \rangle| \in \left[\frac{9134}{1000}, \frac{9136}{1000}\right], \quad \frac{9134}{1000} > 9. \quad (53)$$

Consequently, no maintained continuous history exists from this state under the nominal probe law and declared continuous-time work predicate. For the separately specified continuous-future law in Eq. (52), both original states also have empty viable endpoint capacities: at $T_f = 1/4$ the absolute drive-state pairing exceeds 9 for each exact future trajectory. ¹⁷

Proof. Put $K = E + D$, $\lambda_{\pm} = (5 \pm \sqrt{5})/2$ and $\bar{z} = K^{-1}q = (31/30, -89/60)$. The exact probe trajectory from y is

$$z(t) = \bar{z} + e^{-\lambda_- t} \frac{\lambda_+ I - K}{\sqrt{5}}(y - \bar{z}) + e^{-\lambda_+ t} \frac{K - \lambda_- I}{\sqrt{5}}(y - \bar{z}). \quad (54)$$

The eigenprojectors sum to I and satisfy $KP_{\pm} = \lambda_{\pm}P_{\pm}$, which directly verifies the initial condition and differential equation. Both positive decay rates imply uniqueness by the scalar integrating-factor argument in the two eigenmodes. The explicit rational enclosure of $\sqrt{5}$ and the alternating exponential bounds in Appendix E, applied to the two spectral terms in Eq. (54), give the intermediate coordinate certificate

$$\frac{947856}{10^6} \leq z_0(T) \leq \frac{947858}{10^6}, \quad -\frac{1422185}{10^6} \leq z_1(T) \leq -\frac{1422182}{10^6}. \quad (55)$$

¹⁷Formal counterpart:

Tier33.RelationalHealth.Measurement.HMS06.continuous_future_both_capacities_empty.

At $T = 1/6$ the instantaneous work is $W(T) = |\frac{55}{12}z_0(T) - z_0(T)^2 - 4z_1(T)|$. Interval substitution into this polynomial gives

$$\frac{9134633}{10^6} < W(T) < \frac{9134659}{10^6},$$

which establishes Eq. (53) and rules out maintained exact histories.

For the continuous future, write $z_f = (A + m, -A + m)$. From Eq. (52),

$$A(T_f) = 1 + \frac{1}{8}e^{-3/4} \geq 1, \quad m_x(T_f) = \frac{17}{4} - 4e^{-1/4}, \quad m_y(T_f) = \frac{17}{4} - \frac{9}{2}e^{-1/4}.$$

The elementary inequality $e^{1/4} \geq 1 + 1/4 = 5/4$ gives $e^{-1/4} \leq 4/5$. Moreover, $\langle u_f, z_f \rangle = 6A + \frac{17}{2}m$. For the smaller y common mode,

$$\langle u_f, z_f(T_f; y) \rangle \geq 6 + \frac{17}{2} \left(\frac{17}{4} - \frac{9}{2} \frac{4}{5} \right) = \frac{461}{40} > 9. \quad (56)$$

The x common mode is larger by $\frac{1}{2}e^{-1/4} > 0$, so its future work also exceeds 9. Uniqueness makes these the only respective lawful exact flows. Both continuous-future viable capacities are empty, with no appeal to sampled-time claims. $\square$

The sampled and continuous analyses use different trajectories and different tests of work. The discrete procedure evaluates source work at each recorded step, whereas the exact-flow model requires the instantaneous budget throughout the trajectory. Theorem 11.1 establishes the failure of automatic transfer between these specified conditions. The respective contributions of numerical discretization and the work convention would require further analysis under a common temporal model.

11.3 A survival failure between qualified observations

Endpoint qualification presents a distinct information question even when the continuous-time law is fixed. The following example has qualified beginning and ending states and an interval of constitutive failure between them.

Proposition 11.2 (Qualified endpoints conceal an interior survival failure). *For the exact homogeneous continuous-time law in Eq. (50), take $z(0) = (1, -1)$, constant drive $u = (-9/2, 9/2)$, terminal time $T = \log 5/3$ and interior time $t_* = \log(5/3)/3$. Then $0 < t_* < T < 1$. Both $z(0)$ and $z(T)$ satisfy the original real-profile qualification conditions; at every time in $[0, T]$ the cost and instantaneous work magnitudes are at most 9. Nevertheless*

$$z(T) = (-1, 1), \quad z(t_*) = (0, 0), \quad |J_{01}(z(t_*))| = 0 < 1.$$

Thus the endpoint qualification, whole-interval cost, and instantaneous-work conditions together leave the all-time exchange-survival requirement unresolved. The exhibited trajectory violates that survival requirement at an interior time. ¹⁸

¹⁸Formal counterpart:

Tier33.RelationalHealth.Measurement.HMS06.actual_ode_samples_miss_survival_floor.

Proof. The law preserves the antisymmetric subspace: write $z(t) = (a(t), -a(t))$. Its exact scalar equation is $a'(t) = -3a(t) - 9/2$ with $a(0) = 1$, hence

$$a(t) = -\frac{3}{2} + \frac{5}{2}e^{-3t}.$$

At $T = \log 5/3$ we obtain $a(T) = -1$; at $t_* = \log(5/3)/3$ we obtain $a(t_*) = 0$. Throughout the interval, $-1 \leq a(t) \leq 1$, giving $C(z(t)) = 6a(t)^2 \leq 6$ and $|\langle u, z(t) \rangle| = 9|a(t)| \leq 9$. Both endpoints have edge-flux magnitude 2 and cost 6. At the interior crossing, the edge flux $2a(t_*)$ vanishes, violating the required survival floor 1. This proves a genuine within-flow observation obstruction independent of the comparison between Euler and exact dynamics. $\square$

An exact positive continuous-time control also exists: initial state $(1, -1)$ and constant drive $(3, -3)$ give a stationary trajectory satisfying the continuous-maintenance requirements throughout each horizon $0 \leq T \leq 1$. The stationary case demonstrates that continuous maintenance remains attainable under a suitable drive, while the preceding example locates an actual interior violation under different forcing.

Remark 11.3 (Interpretation of the temporal results). The two findings bear on different aspects of temporal resolution. Theorem 11.1 concerns comparison of maintenance conclusions across different dynamical and work specifications. Proposition 11.2 concerns survival information lost between two qualified observations of one exact trajectory. Physical application calls for measured time references, supported dynamics, and a justified choice of sampled, instantaneous, or cumulative work evaluation.

12 Discussion

The results establish mathematical identification and decision preservation under the specified two-site constitution and observation models. Physiological state imaging remains a proposed metrological application requiring independent physical measurement and validation. The fixed-drive and heterogeneous interventions provide two model-level means of meeting the same maintenance and identification requirements.

12.1 The organizational state as the object of measurement

The state x appearing in the prospective Health proposition raises a central question for measurement science: what determines the state occupied by the organization under study? A scientific specification establishes the relevant type of state, the physical or organizational distinctions represented by its coordinates, and the constitutive requirements governing its continuation. Observations establish which states are consistent with the organization as it is encountered. In some domains, those quantities are directly accessible through established measurements. In a distributed physiological organization, recovering the relevant state can itself be the principal task of a measurement system.

The intended measurand is therefore the organization's state at a specified resolution and under stated physical conditions. An instrument may estimate this state through boundary responses, interactions, or other observable consequences. The measurement result includes the distinctions that

the available evidence resolves and the ambiguities that remain. If an observation y is interpreted through a justified model with uncertainty ε , the resulting state image can be represented as the compatible family

$$\mathcal{X}_y = \{z \in X : \text{Compatible}(z, y; \mathcal{S}, \varepsilon)\}, \quad (57)$$

A singleton image identifies a state within the specified carrier and observation model; a larger family describes the ambiguity that remains under those assumptions. The size of this family depends on both the discriminatory information in the data and the coverage of the model. Informative observations can narrow the family; an incomplete carrier that excludes physically possible states can produce the same apparent precision. Independent examination of the state carrier, measurement response model and physical reference is therefore necessary before a model-identified state can be interpreted as an empirically validated image of the organization. For a particular scientific question, the required information may be available before the full physical state is uniquely reconstructed. The observation-fiber theorems of this paper give an exact model-relative instance: the initial readings specify two candidates, an informative intervention separates them, and the prospective Health result follows for every candidate compatible with the combined evidence.

This is the point at which a future metrological system for physiological state imaging enters the broader Health framework. Such a system is intended to provide evidence about the organization's state, with uncertainties that reflect both measurement limitations and possible inadequacies in the physical model. The scientific interpretation of that state, the laws governing its continuations, and the adequacy requirements are separately specified and tested. Independent measurements and predictive tests must assess whether the selected carrier and observation law adequately represent the physical organization. The particular sensing and inverse-reconstruction methods are implementation questions, with their own calibration and experimental validation responsibilities.

A state image may also be evaluated at several organizational scales. For each scale s , let π_s be an appropriately justified state representation. A scale-specific Health question takes the form

$$H_{\mathcal{S}_s, q_s}(\pi_s(z); d_s, t_s), \quad z \in \mathcal{X}_y, \quad (58)$$

The validity of the map π_s , the coverage of the original state carrier, and the corresponding continuation model are substantive scientific requirements. When every candidate in $\mathcal{X}_y$ yields the same answer to a scale-specific query, that answer is identified relative to those assumptions despite remaining uncertainty about other aspects of the state. The same mathematical framework can therefore operate with states obtained from a forest inventory, engineered-system instrumentation, or a future physiological imaging procedure, while respecting the distinct evidentiary basis of each application.

12.2 Measurement intervention and state-dependent response

The first constructive intervention uses a heterogeneous response rule $u(z) = q - Dz$. This rule is defined for the actual state, so its physical realization requires a mechanism that responds to an absolute local reference. A controller whose only information is the original flux-and-cost indication cannot determine that hidden coordinate from the baseline reading. The observation-only result identifies the corresponding obstruction for a homogeneous one-step flux readout. Section 8.4 gives a fixed-drive procedure resolving the same ambiguity through repeated cost observation. The cost channel and its calibration thereby become part of the measurement-design decision.

The distinction between identifying the original state and preserving its prospective classification is important for state imaging. A measurement can be informative and constitutionally maintained while changing the organization’s subsequent viable responses. The two preserving experiments address that concern for the complete original observation fiber, under different post-probe sensing requirements. Their results establish stable Health classifications under the specified future law and identify the associated changes in response-valued capacity. The two future questions remain anchored to their respective source states.

12.3 Metrological qualification and predictive use

Translating this mathematical experiment into an operational measurement requires independent evidence for the measurand, physical reference, acquisition mechanism, response coefficients, measurement timing, uncertainty, and model discrepancy. Relevant metrological principles address measurement models, traceability and uncertainty-based decisions [6, 7, 8, 9, 10]. The small error bounds proved here characterize a mathematical region of reliable identification; their feasibility for a physical device remains to be assessed by calibration and controlled tests.

The future law presents a further scientific obligation. A physiologically meaningful prospective prediction requires a justified dynamical model for the organization and its context, followed by temporally independent evaluation of predicted continuation outcomes. Mathematical sufficiency of the observations establishes which model-defined answers can be inferred; observational validation establishes how those answers relate to the material system. These responsibilities can be assessed separately and accumulated into a qualified measurement result.

12.4 Generality and the scope of the example

The present two-site model provides a complete and analytically tractable example of a constitutive measurement problem. Its structural information analysis is related to functional observability, experimental design, and partial identification [18, 21, 19, 20, 22, 12, 11]. The contribution concerns the simultaneous satisfaction of identification, constitutional maintenance, and preservation of the prospective property. The two experiments show how the chosen post-intervention observation channel enters these requirements. Exact fiber reconstruction and quantified local bounds make all three obligations inspectable.

13 Conclusion

We have established an exact measurement problem in which states with identical present flux and cost possess different prospective continuation capacities. Their complete observation fiber contains two constitutively qualified states whose responses to a common future law differ in viability. The analysis identifies the hidden state distinction, demonstrates how an informative intervention may alter prospective Health, and constructs preserving measurement procedures using either a heterogeneous flux-readout probe or a fixed-drive repeated-cost observation. The proofs cover the complete exact information fiber, a nontrivial interval of heterogeneous interventions, and a separately analyzed fixed-drive alternative.

The heterogeneous identification procedure extends to complete state families compatible with bounded baseline measurement errors. The fixed-drive comparator also admits a separate local bounded-error guarantee through the cost channel. Uniform inequalities for uncertain state, intervention, and response quantities provide a decision rule, while a nonzero neighborhood of future laws establishes local robustness to changes in the specified future conditions. The resulting guarantees distinguish the ability to perform a constitutionally maintained intervention from the evidence needed to report a prospective Health conclusion. They also distinguish preservation of a Health classification from preservation of the complete response-valued continuation capacity.

Continuous-time investigation adds two complementary temporal findings. The nominal sampled-maintained intervention exceeds an all-time work budget under a separately specified exact-flow model. An independent continuous trajectory retains qualified endpoints while crossing a survival boundary between observations. These results establish the need for explicit dynamical and temporal assumptions when measurements are used to evaluate lawful continuation.

The scientific state x in the Health proposition is both the subject of constitutional interpretation and a potential target of measurement. In a physiological application, a future state-imaging instrument is intended to provide an evidence-supported representation of that state, with its observational resolution and uncertainty. The measured state can then support realization, continuation-capacity and adequacy questions at those organizational scales for which the relevant representations and laws are justified. The series thereby moves from a constitutive definition through contextual and representational obligations and empirical assessment to the conditions for informative, maintained measurement. The present mathematical results define conditions for that inference; physical calibration and independent prospective validation determine its empirical authority.

A Mathematical results and verification scope

The scientific statements are organized by the role they play in the present argument. The constitutive-maintenance model, its sampled histories, and the original prospective viability conditions are the input specification. The full present-observation fiber, intervention-induced reversal, heterogeneous preserving identification, complete baseline-compatible family, and shared-future-law uncertainty are established mathematical results of the preceding formal research. The response-transport proposition and continuous-time specializations clarify the limits of their interpretation. The fixed-drive theorem and its local uncertainty comparison are new mathematical derivations in the present version.

Mathematical result	Verification and evidentiary scope
Two-site maintenance and specified prospective law	Original model; exact definitions and proof in this manuscript
Full exact fiber and prospective separation	Previously verified privately; mathematical argument reproduced here
Measurement reversal and preserving heterogeneous intervention	Previously verified privately; exact examples and proofs reproduced here
Joint uncertainty and local future-law family	Previously verified privately; mathematical bounds summarized with rational enclosure ledger
Fixed-drive, repeated-cost identification	New exact rational proof in this version; no prior Lean certification claimed
Fixed-drive local bounded-error comparison	New rational interval calculation in this version; mathematical construction given in Appendix D
Continuous-time flow and nontransfer	Previously verified privately; dynamics and negative examples explicitly given
Physical acquisition and predictive validity	Empirical requirements; no instrument or outcome validation established here

Formal counterparts and the corresponding source revision are recorded in the theorem footnotes and Appendix F.

B Further exact arithmetic

The original one-step constitution fixes $o = 2$, $s = 1$, $B = 9$, $N = 2$ and $T = 1$. For the shared one-step future, the work pairing uses the source profile of that history. In the two-stage protocol, the future history begins at the actual post-probe profile. The probe and future retain their distinct registered durations and their separate inherited Evolution records.

The two-site observation gives $J = 2A$ and $C = 6A^2 + 2m^2$ for $\phi = (A + m, -A + m)$. Under prescribed $(v + d, v - d)$ the future modes satisfy $A' = (1 - 3h)A + hd$, $m' = (1 - h)m + hv$, and the cost-separation identity of Eq. (28) follows by expanding $2[(m'_+)^2 - (m'_-)^2]$. The exact compensation polynomials are listed in the proof of Theorem 8.2; their rational parameter interval is universal, not a finite enumeration. The interval rules and the necessary threshold separations are summarized in Appendix E.

For the real-time analytic model, $E + D$ has trace 5 and determinant 5, so its eigenvalues are $(5 \pm \sqrt{5})/2$. The continuous-flow certificate uses rational upper/lower bounds for $\sqrt{5}$ and exponential Taylor

remainders. The work violation in Theorem 11.1 is verified over the entire defined physical-state history class through uniqueness. A sample-safe but interior-unsafe trajectory remains a separate, explicit negative control.

C Exact joint-uncertainty enclosure ledger

Conservative bounds for the full joint state, probe, and future-law uncertainty family are reproduced below as rational outer enclosures. Their precision is sufficient for all mathematical decisions and follows from the rules in Appendix E. The two columns quantify over every state, heterogeneous-probe parameter, and shared-future law in the stated Cartesian products around x and y . Source cost, edge floors, probe work and time conditions are bounded by the same rational interval operations. All decisions use exact fractions.

Quantity	X-branch outer interval	Y-branch outer interval
Initial edge flux	[2249/1000, 2251/1000]	[2249/1000, 2251/1000]
Initial constitutive cost	[1543/200, 7723/1000]	[1543/200, 7723/1000]
Probe endpoint cost	[8149/1000, 1021/125]	[8809/1000, 8827/1000]
Probe drive work	[1977/250, 1583/200]	[8741/1000, 2187/250]
Initial future cost	[9486/1000, 9525/1000]	[7892/1000, 7932/1000]
Post-probe future flux	[2077/1000, 2086/1000]	[2098/1000, 2107/1000]
Post-probe future cost	[9206/1000, 9262/1000]	[8180/1000, 8235/1000]
Post-probe future work	[8183/1000, 8196/1000]	[5245/1000, 5258/1000]
Probe response flux	[93/40, 291/125]	[301/125, 2411/1000]

The complete maintenance and prospective-decision certificate is summarized next. The endpoints are terminating rational numbers; their enclosure follows from the full coordinate recurrence in Appendix E. Every source, probe and future-law parameter varies independently in its displayed interval. The displayed intervals provide a self-contained exact separation certificate. The largest

Required quantity	x branch	y branch
Source $J(z)$	[2.249, 2.251]	[2.249, 2.251]
Source $C(z)$	[7.715, 7.723]	[7.715, 7.723]
Probe $J(Pz)$	[2.325, 2.328]	[2.408, 2.411]
Probe $C(Pz)$	[8.149, 8.168]	[8.809, 8.827]
Probe work W_p	[7.908, 7.915]	[8.741, 8.748]
Unprobed future $J(Fz)$	[2.059, 2.066]	[2.059, 2.066]
Unprobed future $C(Fz)$	[9.486, 9.525]	[7.892, 7.932]
Unprobed future work	[8.872, 8.878]	[4.622, 4.628]
Post-probe future $J(F(Pz))$	[2.077, 2.086]	[2.098, 2.107]
Post-probe future $C(F(Pz))$	[9.206, 9.262]	[8.180, 8.235]
Post-probe future work	[8.183, 8.196]	[5.245, 5.258]

Table 1: Complete outward rational enclosure of the source, measurement and future-law inequalities. Displayed decimals are exact fractions with denominator 1000; each interval contains the corresponding result of exact rational interval propagation.

x -branch response is at most 291/125 and the smallest y -branch response is at least 301/125.

Allowing an independent readout error of $1/1024$ on each branch gives

$$\left(\frac{301}{125} - \frac{1}{1024}\right) - \left(\frac{291}{125} + \frac{1}{1024}\right) = \boxed{\frac{999}{12800} > 0}. \quad (59)$$

Moreover, $291/125 + 1/1024 < 341/144 < 301/125 - 1/1024$, so the published threshold lies strictly between the two expanded intervals. This conservative certificate suffices for every separation and classification assertion in the paper. The interval calculus establishes products by all four rational corner inequalities, propagates addition and subtraction with outward-safe operations, and normalizes the original exchange and cost polynomials exactly. The mathematical proofs quantify over every rational member of each specified uncertainty box.

D Exact fixed-drive calculations and rational interval bounds

The fixed-drive calculation uses the complete two-site model stated in the main text. The endpoints and their future responses are obtained by exact fractional arithmetic from Eqs. (13), (21), and (38). For transparent uncertainty accounting, write $I[a, b]$ for a closed rational interval. The sum and difference of two such intervals are enclosed by $I[a + c, b + d]$ and $I[a - d, b - c]$; their product is enclosed by the minimum and maximum of the four corner products ac, ad, bc, bd . These rules are inclusion-preserving for every real or rational input in the given intervals.

Let $\rho = 1/4096$. The independently varying input bounds are

$$\begin{aligned} \|z - x\|_\infty \leq \rho \quad \text{or} \quad \|z - y\|_\infty \leq \rho, \quad |h_p - 1/6| \leq \rho, \\ \|q_p - (4, -2)\|_\infty \leq \rho, \quad |h_f - 1/4| \leq \rho, \quad \|u_f - (29/4, 5/4)\|_\infty \leq \rho. \end{aligned}$$

Apply the interval rules to $P(z) = z + h_p(q_p - Ez)$ and $F(Pz) = Pz + h_f(u_f - E(Pz))$, then to $C(w) = (w_0 - w_1)^2 + w_0^2 + w_1^2$ and the drive/source work pairings. The outer rational bounds in Section 8.5 enclose the resulting expressions. For completeness, the same complete two-phase interval calculation gives In particular, the y -branch future source work lies within $[6.014, 6.027]$.

Quantity	x branch	y branch
Post-probe flux	[2.123, 2.127]	[2.123, 2.127]
Post-probe cost	[7.047, 7.063]	[6.767, 6.787]
Probe source work	[7.247, 7.253]	[6.247, 6.253]
Future flux after probe	[2.026, 2.037]	[2.026, 2.037]
Future cost after probe	[9.773, 9.828]	[8.287, 8.345]
Future source work after probe	[9.557, 9.568]	[6.014, 6.027]

Table 2: Fixed-drive certificate on the full independent rational error boxes, using denominator-1000 outer intervals.

Both source neighborhoods retain initial cost below 9 and exchange above 2. Both probe endpoints have cost below 9, flux above 2, and source work below 9. The probe duration satisfies $0 < h_p < 1/3$, so the original stability, survival and recovery conditions hold. The x -branch future cost exceeds 9, whereas the y -branch future cost is below 9 with qualifying future flux and work. Finally,

$$6.787 + 1/1024 < 69/10 < 7.047 - 1/1024,$$

which makes the post-probe cost threshold robust under the stated readout error. These results are direct rational-interval consequences of the displayed model and uncertainty bounds.

E Self-contained proofs of uncertainty and temporal results

The following exact interval calculus establishes decision inequalities uniformly across the stated uncertainty regions, including every rational parameter choice in the sampled model.

For closed rational intervals $I = [a, b]$ and $J = [c, d]$, define

$$I + J = [a + c, b + d], \quad I - J = [a - d, b - c], \quad I \cdot J = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)]. \quad (60)$$

For an interval containing zero, $|I| = [0, \max(|a|, |b|)]$; otherwise take the minimum and maximum endpoint magnitudes. If $0 < a \leq b$, its reciprocal interval is $[1/b, 1/a]$. Each is an inclusion enclosure for all real operands in its two input intervals, by the monotonicity of addition, the four extreme corner products, and positivity of reciprocal. Structural induction on an arithmetic expression establishes enclosure of the entire expression, including dependencies that appear more than once. The interval need not be sharp to prove a strict inequality.

The two-site operator is $E(z_0, z_1) = (2z_0 - z_1, 2z_1 - z_0)$, with $J(z) = |z_0 - z_1|$ and $C(z) = (z_0 - z_1)^2 + z_0^2 + z_1^2$. The full heterogeneous and fixed-drive sampled updates, respectively, are

$$P(z) = z + h(q - \text{diag}(\eta_0, \eta_1)z - Ez), \quad \tilde{P}(z) = z + h(q - Ez), \quad F(z) = z + h_f(u_f - Ez). \quad (61)$$

The exact Cartesian input intervals are

$$\begin{aligned} \rho &= 1/4096, & Z_i &\in [z_{*,i} - \rho, z_{*,i} + \rho], & H &\in [1/6 - \rho, 1/6 + \rho], \\ Q_0 &\in [55/12 - \rho, 55/12 + \rho], & Q_1 &\in [-4 - \rho, -4 + \rho], \\ \eta_0 &\in [1 - \rho, 1 + \rho], & \eta_1 &\in [0, \rho], \\ H_f &\in [1/4 - \rho, 1/4 + \rho], \\ U_{f,0} &\in [29/4 - \rho, 29/4 + \rho], & U_{f,1} &\in [5/4 - \rho, 5/4 + \rho], \end{aligned} \quad (62)$$

with $z_* \in \{x, y\}$, and independent readout error $[-1/1024, 1/1024]$. Apply Eq. (60) in this exact coordinate order: for $i \in \{0, 1\}$ and $j = 1 - i$,

$$\begin{aligned} U_i &= Q_i - \eta_i Z_i, \\ P_i &= Z_i + H(U_i - 2Z_i + Z_j), \\ R_i &= P_i + H_f(U_{f,i} - 2P_i + P_j). \end{aligned}$$

(63)

Use the same recurrence with Z in place of P to bound the unprobed future. The complete decision expressions are

$$\begin{aligned} J(P) &= |P_0 - P_1|, & C(P) &= (P_0 - P_1)^2 + P_0^2 + P_1^2, \\ W_p &= |U_0 Z_0 + U_1 Z_1|, & J(R) &= |R_0 - R_1|, \\ C(R) &= (R_0 - R_1)^2 + R_0^2 + R_1^2, & W_f &= |U_{f,0} P_0 + U_{f,1} P_1|. \end{aligned} \quad (64)$$

For the fixed-drive comparator set $\eta_0 = \eta_1 = 0$ and $Q_0 \in [4 - \rho, 4 + \rho]$, $Q_1 \in [-2 - \rho, -2 + \rho]$; every other interval remains the same.

One explicit worked propagation makes the enclosure procedure inspectable. For the x -branch,

$$\begin{aligned} Z_0 &= [11/8 - \rho, 11/8 + \rho], & Z_1 &= [-7/8 - \rho, -7/8 + \rho], \\ D_0 &:= Q_0 - (2 + \eta_0)Z_0 + Z_1, & P_0 &= Z_0 + H D_0. \end{aligned}$$

Since $Z_0 > 0$ and the entire interval D_0 is negative, its lower and upper bounds are respectively

$$\begin{aligned} d_- &= (55/12 - \rho) - (3 + \rho)(11/8 + \rho) - 7/8 - \rho, \\ d_+ &= (55/12 + \rho) - (3 - \rho)(11/8 - \rho) - 7/8 + \rho, \\ p_- &= 11/8 - \rho + (1/6 + \rho)d_-, \\ p_+ &= 11/8 + \rho + (1/6 - \rho)d_+. \end{aligned}$$

Direct rational evaluation gives $1304949/10^6 \leq p_- \leq p_+ \leq 1306161/10^6$. The remaining exact coordinate evaluations, using the unrounded expressions in Eq. (63), are bounded by the following denominator- 10^6 ledger:

Coordinate	x interval times 10^6	y interval times 10^6
P_0	[1304949, 1306161]	[971596, 972849]
P_1	[-1021455, -1020177]	[-1437999, -1436945]
R_0	[2208062, 2212086]	[1937168, 1941320]
R_1	[126189, 130774]	[-165432, -160928]

These rational inclusions and Eqs. (60), (64) yield every row in Tables 1 and 2 when the appropriate drive intervals are substituted. Each bound is evaluated over the entire parameter box, not from a finite set of point samples. For the heterogeneous experiment, the state radius and all probe perturbations are at most $\rho = 1/4096$; the second relaxation coefficient belongs to $[0, \rho]$. The future duration and drive each vary by ρ . A separate response error is at most $1/1024$. For the fixed-drive experiment the same radius applies to its step, two drive coordinates, initial state and future law. These are independent coordinates in their full Cartesian products.

Verified quantity	Negative x region	Positive y region
Heterogeneous post-probe cost	[8.149, 8.168]	[8.809, 8.827]
Heterogeneous future cost	[9.206, 9.262]	[8.180, 8.235]
Heterogeneous flux, including error	[2.324, 2.330]	[2.407, 2.413]
Fixed-drive post-probe cost, including error	[7.046, 7.064]	[6.766, 6.788]
Fixed-drive future cost	[9.773, 9.828]	[8.287, 8.345]

Table 3: Coarse outward decimal summaries of exact rational enclosures computed by the interval operations. All decisions are checked on exact fractions; the table is provided for readability.

All quantitative bounds in Table 3 are outer enclosures obtained by applying Eq. (60) to Eq. (61) and the cost and work polynomials. The strict margins prove qualification, maintenance and the required separation for the complete stated parameter regions.

For the analytic continuous-time probe, use the explicit rational enclosure

$$\frac{2236067977499}{10^{12}} < \sqrt{5} < \frac{2236067977500}{10^{12}}, \quad \left(\frac{2236067977499}{10^{12}} \right)^2 < 5 < \left(\frac{2236067977500}{10^{12}} \right)^2. \quad (65)$$

It is combined with the elementary alternating-series enclosure

$$\sum_{k=0}^{11} \frac{(-s)^k}{k!} \leq e^{-s} \leq \sum_{k=0}^{10} \frac{(-s)^k}{k!} \quad (0 \leq s \leq 1). \quad (66)$$

These bounds follow from the alternating-series remainder inequality. Exponential monotonicity then permits substitution of the outer rational bounds for $\lambda_{\pm}/6$ into Eq. (54). The resulting interval encloses the unique exact-flow probe work. This procedure proves the displayed inequalities without floating-point rounding assumptions.

For the fixed continuous-future law, the antisymmetric and common-mode coordinates satisfy $A(t) = 1 + \frac{1}{8}e^{-3t}$ and $m_x(t) = \frac{17}{4} - 4e^{-t}$, $m_y(t) = \frac{17}{4} - \frac{9}{2}e^{-t}$. At $t = 1/4$, the elementary bound $e^{-1/4} \leq 4/5$ gives

$$\langle u_f, z_f(t) \rangle = 6A(t) + \frac{17}{2}m(t) \geq 6 + \frac{17}{2} \left(\frac{17}{4} - \frac{9}{2} \cdot \frac{4}{5} \right) > 9.$$

Both nominal continuous futures therefore violate the declared instantaneous-work condition at their terminal time. The eigenprojector expression in Eq. (54), together with Eq. (66), provides rational bounds for the distinct continuous-probe calculation.

F Formal verification provenance

The formal counterparts identified in the theorem footnotes are maintained at scientific source commit a64e66fdbcfed8d14c7ece42f75e674a0ca07934. The Lean toolchain is pinned to `leanprover/lean4:v4.28.0-rc1`, and the Mathlib revision is 5352afccd6866369be9de43f5b7ec47203555f44. The research repository is private and is open for formal review through professional collaboration.

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